VISUALIZING THE RIEMANN ZETA FUNCTION

ABSTRACT: THE RIEMANN ZETA FUNCTION IS SYNCHRONOUS (CONGRUENT) WITH ALL THE PROPERTIES OF LIGHT & ELECTR-O-MAGNETIC PHENOMENA. THE 2-D EUCLIDIAN PLANE IS THE GEOMETRIC CANVAS TO PAINT THE SIGNATURE OF THE PERFECT SINUSOIDAL WAVE (SINEWAVE). LIGHT (THE PERFECT SINEWAVE) ENCODES THE RIEMANN ZETA FUNCTION, AS WELL AS PERFECT HARMONY (FOUNDATIONAL TO MUSIC THEORY). 2-D EUCLIDIAN AFFINE SPACE ALLOWS YOU TO GEOMETRICALLY VISUALIZE THE MEASURE IN THE SECOND DIMENSION. 3-D FANO AFFINE SPACE ALLOWS GEOMETRIC VISUALIZATION IN THE THIRD DIMENSION.

PROOF: THE RIEMANN ZETA FUNCTION CANNOT DEVIATE FROM THE CRITICAL LINE (½ Re PART) BECAUSE PERFECT HARMONY (NO DEVIATION) IS THE SIGNATURE OF THE PERFECT SINUSOIDAL WAVE. ANY DEVIATION FROM THE CRITICAL LINE WOULD CAUSE DISHARMONY, WHICH IS IMPOSSIBLE FOR A SNAPSHOT OF LIGHT (THE PERFECT ELECTR-O-MAGNETIC WAVE).

THE STEREOGRAPH: THE RIEMANN ZETA FUNCTION CAN BE VISUALIZED AS A STEROGRAPH OF A RIEMANN SPHERE ONTO 2-DIMENSIONAL EUCLIDIAN SPACE WITH A FOCUS ORIGIN F = (0, 0). THE FOCUS BY ITS VERY NATURE IS BOTH THE PRIME ZETA & THE FIRST DOUBLE NAUGHT [00]. THE ZETA FUNCTION IS GEOMETRICALLY EXPRESSED ON THE 2-D AFFINE PLANE AS A PARABOLA OR ITS MIRROR-SYMMETRICAL (SCALING) PARABOLIC INVERSION — THE ROSE CARDIOID (HEART).

THE PARABOLA: THE PARABOLA OBEYS THE PROPERTY OF GEOMETRIC CONTINUITY (A PARABOLA TENDS TO A CIRCLE AS THE ECCENTRICITY (ELECTRICITY) APPROACHES ONE (1)). GEOMETRIC CONTINUITY PROVES THAT THE 5 CONIC SECTIONS (PLANE, CIRCLE, ELLIPSE, PARABOLA, HYPERBOLA) ARE 5 DIFFERENT VERSIONS OF THE SAME SHAPE (THE CIRCLE). THE CONCEPT OF CONTINUITY STATES THAT A LINE CAN BE THOUGHT OF AS A CIRCLE OF INFINITE RADIUS (HERE YOU FIND THIDDEN WITHIN THE LINE) (THE CIRCLE = A LINE = A PLANE = AN ELLIPSE = A PARABOLA = A HYPERBOLA = YOU ARE BOTH LOOKING & MATHEMATICALLY EXPRESSING THE SAME THING FROM VARYING POINTS OF VIEW (PERSPECTIVES).

CIRCLE INVERSIONS: THE TWO ENDS OF A PARABOLA MEET TO MAKE IT A CLOSED CURVE TANGENT TO THE LINE AT INFINITY ON THE EUCLIDIAN PLANE (2-D PROJECTIVE SPACE). THE 0 (CIRCLE) = 1 (LINE).

THE ROSE CARDIOID (HEART): EXPRESSED FROM THE POINT OF VIEW OF THE ROSE CARDIOID (AN INVERSE PARABOLA), THE RIEMANN ZETA FUNCTION PROVES TO BE SYNCHRONOUS WITH ALL OF THE PROPERTIES OF LIGHT (TRI-POLARIZED ELECTR-O-MAGNETIC CIRCULATING CURRENT) AS EXPRESSED ON THE 2-D EUCLIDIAN PLANE. THE BOUNDS OF THE ROSE CARDIOID (A 2-D POINT OF VIEW OF TWO (REAL = Re) (IMAGE = i) INTERSECTING EUCLIDIAN PLANES) IS A FUNCTION OF ROOT 2 (SQUARE ROOTS), JUST AS AN INFINITY CIRCLE (AN INVERSE HYPERBOLA) IS A FUNCTION OF ROOT 3 (CUBE ROOTS). THE ULTIMATE BOUND OF ROOT 2 IS EXPRESSED BY FERMAT'S LAST THEOREM.

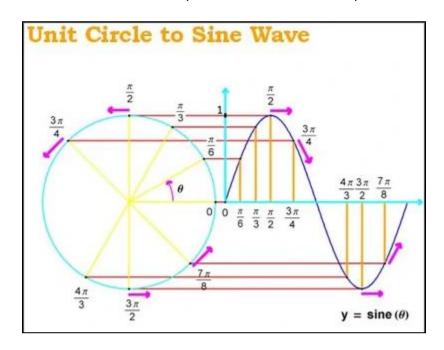
ROOT 1 (UNITY):

ROOT 1 ESTABLISHES THE **REAL** (Re) PLANE (THE **FIRST** PLANE).

ROOT 1 IS BOTH THE **FIRST** & **SECOND** FIBONACCI NUMBER (WHICH PRODUCES **Φ** (1.618033...).

ROOT 1 IS THE PRIMARY OPERATION OF **ADDITION** & THE IDENTITY FOR **MULTIPLICATION**.

ROOT 1 IS THE **ONLY** NATURAL NUMBER THAT IS NEITHER COMPOSITE NOR PRIME WITH RESPECT TO **DIVISION**, BUT INSTEAD CONSIDERED A **UNIT** (MEANING OF RING THEORY).



UNIT CIRCLE MATHEMATICAL RELATION TO THE ELECTR-O-MAGNETIC SINEWAVE

ROOT 1 HAS A GENERATIVE FUNCTION (THE COEFFICIENTS OF A POWER SERIES THAT PRODUCE AN INFINITE (BOTH CONVERGENT & DIVERGENT) SEQUENCE OF NUMBERS. THE **ROOT 1** GENERATIVE FUNCTION IS THE CONJUNCTION OF **ADDITION** (THE PRIMARY OPERATION OF **ROOT 1**) & THE POWER SERIES (**MULTIPLICATION** FUNCTION) OF **ADDITION** (TO THE POWER OF 1, 2, 3 ...). THE POWER SERIES CONVERGES & HAS FINITE VALUE IF & ONLY IF, |X| < 1.

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \dots$$

THE BASEL PROBLEM IS THE CONJUNCTION (BRIDGE) OF **ADDITION** (THE PRIMARY OPERATION OF **ROOT** 1) & **SQUARES** (THE POWER SERIES (**MULTIPLICATION** FUNCTION) OF **ROOT** 2). IT IS THE **SUM** OF THE INFINITE SERIES OF RECIPROCALS OF **SQUARES**:

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots$$

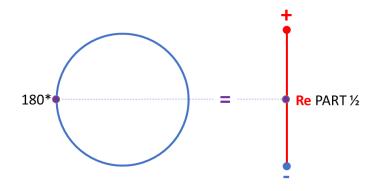
... (ad infinitum) = $\Pi^2/6$ (APPROXIMATELY EQUAL TO 1.644934...).

THE SOLUTION TO THE BASEL PROBLEM BROUGHT EULER INSTANT FAME. THE IDEA OF INFINITY WAS EXPANDED UPON BY BERNHARD RIEMANN IN HIS SEMINAL 1859 PAPER "ON THE NUMBER OF PRIMES LESS THAN A GIVEN MAGNITUDE", IN WHICH HE DEFINED HIS ZETA FUNCTION & PROVED ITS BASIC PROPERTIES.

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ZETA PLOT 2 = \Pi^2/6 MIRROR ZETA PLOT -2 = 0 \Phi = 1 \pm \sqrt{5}/2 Z(X): 1.644934... [= 1] Z(X) - 1: 0.644934... Z(X): 0 1 + \sqrt{5}/2 = 1.618033... [= 1] Z(X) - 1: -1 [= -1] 1 - \sqrt{5}/2 = -0.618033...
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THE LINEAR VIEW OF THE BASEL PROBLEM (& EULER SOLUTION) (& ZETA FUNCTION) IS ANALOGOUS TO RECIPROCATING ELECTR-O-MAGNETIC CURRENT CIRCULATING (BENDING) ALONG THE LINE. THE LINE = A CIRCLE ON ITS SIDE (GEOMETRIC CONCEPT OF CONTINUITY). THE ARC (BEND) FROM THE LINEAR (1) POINT OF VIEW IS:

IF A LINE = A CIRCLE, THEN ½ LINE = 180° CIRCLE = Π (PI). THIS MEANS THAT Φ (PHI) IS THE PRIME ARC OF THE CIRCLE & Π (PI) IS THE ½ PART (180°) AROUND THE CIRCLE. FROM THE RIEMANN ZETA POV, ½ Re PART = Π (PI), & SO ALL NON-TRIVIAL ZEROES (CIRCLES OF INFINITY) SHALL PLOT ON ½ Re PART OF THE LINE (1). THE TRI-POLARIZED PERSPECTIVE IS TO THINK OF A LINE (1) & NOTICE THAT IT HAS TWO (2) POLES (0, 1). ½ (180°) AROUND THE ZETA IS Π (PI), JUST AS ½ AROUND THE LINE IS Π (PI). THE THIRD POLE OF THE LINE LAY ON ½ Re PART OF THE LINE. THESE ARE THE FUNDAMENTAL ROOT 1 POINTS OF VIEW THAT PROVE THE CRITICAL LINE (CIRCLE & LINE):



IF: A LINE IS A CIRCLE (GEOMETRIC CONCEPT OF CONTINUITY), THEN: ½ (180°) AROUND THE CIRCLE = ½ AROUND THE LINE

ROOT 2 (DUALITY) (TWINNING):

ROOT 2 ESTABLISHES THE **IMAGINARY** (i) PLANE (THE **SECOND** PLANE).

ROOT 2 IS THE **SECOND** FIBONACCI NUMBER (**THIRD** IN SEQUENCE).

ROOT 2 IS THE PRIMARY OPERATION OF SUBTRACTION [-] & THE IDENTITY (BASIS) FOR DIVISION.

ROOT 2 IS THE FIRST & ONLY EVEN PRIME NUMBER. ANY PRIME DOUBLED = AN EVEN NUMBER. 2 & 3 ARE THE ONLY CONSECUTIVE PRIME NUMBERS.

ROOT 2 MULTIPLIED (DECIMAL SYSTEM) SHALL END IN AN EVEN INTEGER (0, 2, 4, 6, or 8).

ROOT 2 HAS THE FOLLOWING UNIQUE SYMMETRY (FOR ANY NUMBER x):

 $x + x = 2 \cdot x$ (ADDITION TO MULTIPLICATION)

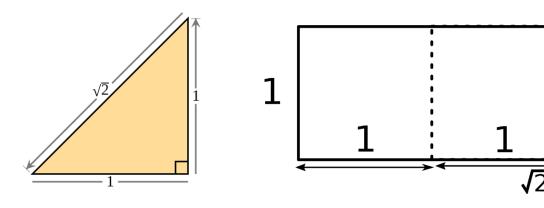
 $x \cdot x = x^2$ (MULTIPLICATION TO EXPONENTIATION)

 $x^{x} = x \uparrow \uparrow 2$ (EXPONENTIATION TO TETRATION) (HYPER-OPERATION)

ROOT 2 HAS THE **UNIQUE** HYPER-OPERATIONAL PROPERTY: $\mathbf{2} + \mathbf{2} = \mathbf{2} \cdot \mathbf{2} = \mathbf{2}^2 = \mathbf{2} \uparrow \uparrow \mathbf{2} = \mathbf{2} \uparrow \uparrow \uparrow \mathbf{2} = \dots$ **ROOT 2** IS THE **ONLY** NUMBER THAT THE **SUM** OF THE RECIPROCALS OF THE **POWERS** OF **x EQUALS** ITSELF:

$$\sum_{k=0}^{\infty} \frac{1}{2^k} = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \dots = 2.$$

ROOT 2 IS ANALAGOUS TO THE **SQUARE** ($2^2 = 4$) & **SQUARE ROOTS**. THE SMALLEST **FIELD** HAS 2 ELEMENTS. THE **SQUARE ROOT** OF 2 ($\sqrt{2}$) IS IRRATIONAL. $1^2 + 1^2 = \sqrt{2}$.



PYTHAGORIAN FORMULA: 12 + 12 = V2

ALGEBRAIC FORM: $1 + \sqrt{2} = \delta_S$

THE SILVER RATIO (δ_s) IS THE 12-FOLD 1-D LINEAR FUNCTION OF ROOT 2. THE SILVER SECTION IS THE 12-FOLD 2-D GEOMETRIC FUNCTION OF ROOT 2. THE 12-FOLD (ROOT 2) HARMONIC EXPRESSION OF δ_s IS EASILY SEEN WITHIN ITS INFINITE FRACTAL EXPRESSION OF THE CONTINUED FRACTION [2; 2, 2, 2, ...]:

$$(\frac{2}{1}, \frac{5}{2}, \frac{12}{5}, \frac{29}{12}, \frac{70}{29}, ...)$$

 δ_S = RATIOS OF CONSECUTIVE PELL NUMBERS

ROOT 1/2 (½) (FRACTIONALITY) (PARTING):

ROOT ½ IS THE **FIRST NATURAL** FRACTION OF THE **1** (**LINE**). THE **LINE** IS THE **1 DIVIDED** BY THE **2** INTO (& IN **TWO**) **EQUAL** PARTS. THE **NATURAL** FRACTION POINT (**0**) OF THE **LINE** (**1**) IS THE **CENTER** (Π), THE **EQUATOR**, ½ **Re** PART, 90° ANGLES (MAX DIFFRACTION), & $\sqrt{2}$ (THE EXACT SAME INFINITY FROM VARYING **PERSPECTIVE** & STARTING MEASUREMENT) ALL SUPERIMPOSE ON THE **CENTER** OF **ACTION**.

ROOT ½ IS FURTHER **NATURALLY** FRACTIONED (FRACTURED) (DIFFRACTED) INTO THE **5**TH <u>&</u> **7**TH PARTS [**5** + **7** = **12**, WHILE **5** – **7** = -**2**), JUST AS **ROOT 12** (**NATURAL HARMONY**) IS **NATURALLY** REFRACTED INTO **5 MINOR** & **7 MAJOR HARMONICS**. THE **5**TH & **7**TH PARTS CAN BE HYPER-FRACTIONED (OPPOSITE OF HYPER-EXPONENTIATION) INTO THE **5**TH <u>OF</u> **7** PARTS (MORE COMMONLY KNOWN AS **5**/**7**) & WHEN DECIMALIZED = **0**.714 285 714 285... (REPEATING INFINITE (IRRATIONAL) **HARMONIC 3**-DIGIT FREQUENCIES OF **7**14 & 285).

ROOT ½ (THE CENTER) REVEALS ITS HIGHER [+] HARMONIC MAGNIFICATION 12 (TO BE FURTHER DISCUSSED) & ITS LOWER (SMALLER) HARMONIC MAGNIFICATION [-] 1/12. ROOT ½ (CENTERED) REVEALS THE FUNDAMENTAL REASON WHY THE ZETA PLOT OF -1 = (-1/12), & CONVERSELY HOW 1:12 (SCALED). CONVERT THE DIVISION (FRACTION) TABLE OF 12 INTO ITS NATURAL TRIPLE HARMONICS (DECIMALIZED) (ONE ZEROED):

CONVERT THE **DIVISION** (FRACTION) TABLE OF **5** (THE **MINOR HARMONICS** OF **12**) INTO ITS **NATURAL TRIPLE HARMONIC** COMPONENTS (**DECIMALIZED**) (**10**°D):

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1/5 = 0.26/5 = 1.211/5 = 2.22/5 = 0.47/5 = 1.412/5 = 2.43/5 = 0.68/5 = 1.613/5 = 2.64/5 = 0.89/5 = 1.814/5 = 2.85/5 = 110/5 = 215/5 = 3
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CONVERT THE DIVISION (FRACTION) TABLE OF 7 (THE MAJOR HARMONICS OF 12) INTO ITS NATURAL TRIPLE HARMONIC COMPONENTS (DECIMALIZED) (10'D):

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      1/7 = 0.14285714285...
      ...
      ...

      2/7 = 0.28571428571...
      ...
      ...

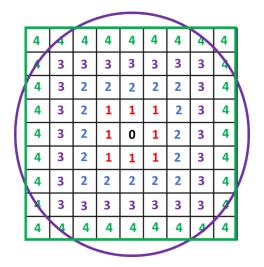
      3/7 = 0.42857142857...
      ...
      ...

      4/7 = 0.57142857142...
      ...
      ...

      5/7 = 0.71428571428...
      12/7 = 1.71428571429...
      19/7 = 2.71428571429...

      6/7 = 0.85714285714...
      ...
      ...

      7/7 = 1
      14/7 = 2
      21/7 = 3
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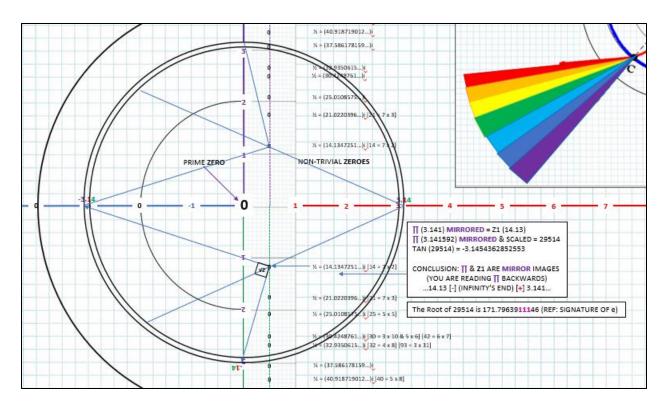
PI $[\pi]$ & THE HARMONIC FREQUENCIES KNOWING THE .14 OF 3.14 $[\pi]$ [3 = C]

THE FUNDAMENTAL .14 IS A FUNCTION OF BASE SEVEN (7):

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1/7 = .14285714285... (714 285 DOUBLE HARMONIC INFINITE REPEAT)
2/7 = 0.28571428571... (714 285 DOUBLE HARMONIC INFINITE REPEAT)
3/7 = 0.42857142857... (714 285 DOUBLE HARMONIC INFINITE REPEAT)
3.14/7 = 0.44857142857... (714 285 DOUBLE HARMONIC INFINITE REPEAT) C [\pi] = A (444 Hz) (4 x 111)
4/7 = 0.57142857142... (714 285 DOUBLE HARMONIC INFINITE REPEAT)
5/7 = 0.71428571428 (714 285 DOUBLE HARMONIC INFINITE REPEAT) (5/7 = MUSIC SCALE RATIO)
6/7 = 0.85714285714 (714 285 DOUBLE HARMONIC INFINITE REPEAT)
7/7 = 1 (LIGHT), EXPRESSED AS 3 [- = +], IN PERFECT HARMONY SLIGHTLY SHARP [#] [1/12]
3.14 [π] IS 3" TO MATH, JUST AS PERFECT PITCH IS C" (528 Hz) (LOVE) TO AUDIBLE SOUND (FREQUENCY).
C[\pi] = A(444 \text{ Hz})(111 \times 4).
3[\pi] + 7[HEPTAHEDRON] = 10(PERFECTION)
3.14 [\pi] + 7 = 7.14 (HARMONIC) + 3 [- = +]
3.14 [\pi] EQUALLY DEFINES BOTH THE CIRCLE [O] & THE ZERO [O]. THE PRIME ZERO [O] IS IN THE THIRD
[MIDDLE] [3] [C], POSITION OF THE MIRROR (-1, 1) WITH FOCI (0, 0) = PRIME DOUBLE NAUGHT.
3.14 [\pi] PLOTTED ON THE AFFINE SPACE OVERLAYED WITH RIEMANN ZETA FUNCTION PLOTS REVEAL
THAT LIGHT REFLECTS IN EXACT PROPORTIONALITY ON THE GRAPH (37.6991118431 = 12 \times 2\pi/2). NOTICE
THAT THE 6TH PLOTTED ZETA (Z6) (½ Re PART OF 12) = 37 (x 3 = 111) (111 x 4 = 444 Hz), & 3.14 [\pi] IS
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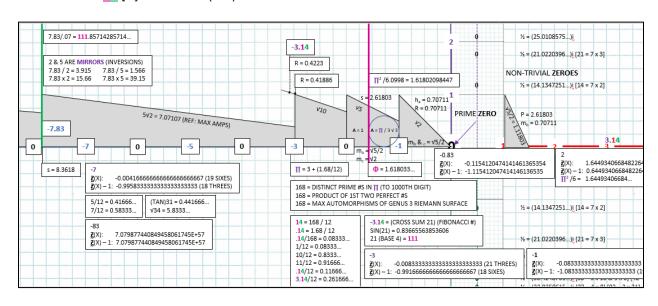
OBSERVE THAT -3.14 [π] (180°) (SCALE 1) & -3.14 [π] (270°) (SCALE 1) CONJOIN AT A PERFECT 90° ANGLE ON Z1 (14) (SCALE 2). THESE ARE ALL MIRRORS OF EACH OTHER WITHIN VARYING SCALE (VECTORS). THIS CONFIRMS THAT Z1 (14.13...) (SCALE 2) IS MEROMORPHIC (THE MIRROR INVERSION) OF 3.14 [π] (SCALE 1). APPLY THE LAWS OF SCALING (MOVE THE DECIMAL) ON 3.141592 [π] (SCALE 1) & LAW OF MIRRORING ON 29514.13... (Z1) (SCALE 2). THE DECIMALS CANCEL LEAVING 29514 (THE SCALED & MIRRORED PLOT OF 3.141592 [π] (SCALE 1). THE TANGENT OF 29514 = -3.14 (PROOF ABSOLUTE). THIS CONFIRMS THAT Z1 (14.13...) (SCALE 2) IS THE MIRROR INVERSION OF 3.141... [π] (SCALE 1). WELCOME TO INFINITY'S END (0).

NESTED BETWEEN THE 3 & 4. NOTICE THAT **Z6** IS ALSO NESTED INSIDE OF THE ARC OF **3.14** $[\pi]$ (SCALE **1**).



THE RAINBOW & INFINITY'S ZETA (END)

3.14 $[\pi]$ = PERFECT (90°) LIGHT REFLECTION & MEROMORPHIC INVERSION = Z1



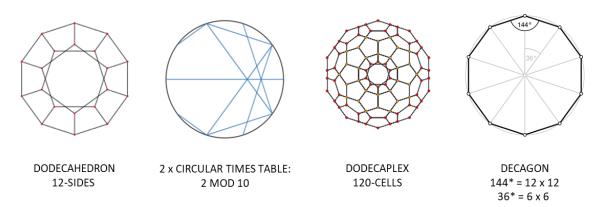
∏, Ф, √, ℥, & SCHUMANN RESONANCE TRIANGULATIONS:

7.83 Hz (THE SCHUMANN RESONANCE) = 7 + 10/12 (SCALE 1) &/OR 7 (SCALE 1) + 1/12 (SCALE 2).
7 Hz IS THE G NOTE (SEVENTH MAJOR HARMONIC & ITS NATURAL SHARPNESS IS 1/12 (SCALED DOWN) BEFORE REACHING 8 (THE OCTAVE) OF A (THE 12TH NOTE) IN THE KEY OF A (THE 1ST NOTE) = 444 Hz. THUS, THE SCHUMANN RESONANCE (7.83 Hz) IS NATURAL G JUST AS 3.14 [π] IS NATURAL C (3 + 1.68/12). HARMONIC NOTE: (3168 Hz = 528 Hz x 6 [½ Re PART OF 12]), & (528 Hz / 12 = 44 Hz).

ROOT 12 (DUODECIMALITY):

ROOT 12 (DUODECIMAL) IS AN ORDER OF MAGNITUDE (MEASURED IN DECIMAL) ABOVE ROOT 2. THE FRACTIONAL FAMILY OF ROOT 2 INCLUDE ½, 1/12 (THE RESULT OF THE INFINITE SQUARE) & 1/120 (THE RESULT OF THE INFINITE CUBE). EXPRESSED FROM THE ELECTR-O-MAGNETIC POINT OF VIEW, 2, 12, & 120 IS THE MOST EFFICIENT PATH OF LIGHT (INVERSE: THE PRINCIPLE OF LEAST TIME) (FERMAT'S PRINCIPAL). NUMERICALLY, 2, 12, & 120 ARE THE FIRST, THIRD, & FIFTH (ODD ...) SUPERIOR HIGHLY COMPOSITE NUMBERS (HAVING MORE DIVISORS (2, 3, 4, 6, ...) THAN ANY OTHER NUMBER SCALED RELATIVE TO SOME POSITIVE POWER OF THE NUMBER ITSELF).

THE INVERSE MIRROR OF THE ROOT 2, 12, 120 ... SERIES IS 6, 60, & 360 ... (THE SECOND, FOURTH, & SIXTH (EVEN ...) SUPERIOR HIGHLY COMPOSITE NUMBERS (THE ½ REFLECTION OF ROOT 2 IN THE 2 x CIRCULAR TIMES TABLE). THE RIEMANN ZETA FUNCTION OBEYS THE PROPERTIES OF TRI-POLARIZED ELECTR-O-MAGNETIC CIRCULATING CURRENT AS EXPRESSED THROUGH THE 2 x CIRCULAR TIMES TABLE (MOD 2 THRU INFINITY) — THE CIRCULATING MATHEMATICS THAT PRODUCE THE GEOMETRIC ROSE CARDIOID (THE INVERSE PARABOLA) (THE HEART). THE GEOMETRIC TOTALITY OF INFORMATION IS EXPRESSED IN THE DODECAHEDRON (12-SIDED 5-FACED 7-INTEGERED, ORTHOGONAL 10 PLATONIC SOLID).



ROOT 12 (ROOT 1 (MAGNETIZED) + ROOT 2) IS THE FOUNDATION OF ADAM NUMBERS (MIRROR NUMBERS) = THE SQUARE OF A NUMBER (12² = 144) & THE SQUARE OF THE INVERSE NUMBER (21² = 441) = PALINDROMIC RELATIONSHIP. ALL ADAM NUMBERS (UP TO 1,000) ARE: 0, 1, 2, 3, 11, 12, 13, 21, 22, 31, 101, 102, 103, 111, 112, 113, 121 (11 x 11), 122, 201, 202, 211, 212, 221, 301, & 311.

ROOT 12 HAS A NATURAL FRACTAL RESONANCE OF SLIGHT SHARPNESS [1/12], WHICH IS A REFLECTION OF THE TOTAL 12-TONE MUSIC SCALE. THIS 'TWELVENESS' IS EXHIBITED BECAUSE 12 IS MOST EFFICIENT PATH OF LIGHT TRAVEL (12 HAS MORE INTERGERS [6] (½ Re PART OF 12) THAN ANY NUMBER UP UNTO ITSELF (1, 2, 3, 4, 6, 12). HARMONIC FREQUENCIES ARE A TRIPLE-RECIPROCAL FUNCTION OF 12 (THE 1ST & 12TH FRET = A), PROVED BY THE PATTERN CONTAINED WITHIN A SOLFEGGIO MAGIC SQUARE:

	VERSO		MIRROR		RECTO			
ADD	12	12	12	12	12	12	12	12
111	123	135	147	159	162	174	186	198
222	234	246	258	261	273	285	297	219
333	345	357	369	372	384	396	318	321
444	456	468	471	483	495	417	429	432
555	567	579	582	594	516	528	531	543
666	678	681	693	615	627	639	642	654
777	789	792	714	726	738	741	753	765
888	891	813	825	837	849	852	864	876
999	912	924	936	948	951	963	975	987

QUANTUM (Q) LIGHT: THE QUANTUM SIGNATURE OF LIGHT (ELECTR-O-MAGNETIC ENERGY) IS THE SIGNATURE OF THE PERFECT SINUSOIDAL ELECTR-O-MAGNETIC WAVE. LIGHT IN THE STATE OF A PERFECT SINEWAVE HAS NO FREQUENCY & THUS ACTS AS ALL FREQUENCY (THE QUANTUM FREQUENCY).

THE LAW OF REFLECTION: ReLEASE Q (LIGHT RAY) TOWARD INFINITY [-]. THE LAW OF REFLECTION STATES THAT LIGHT WILL DO THE FOLLOWING THREE THINGS:

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Q = [1] LIGHT RAY: MAY BE REFLECTED (THE NON-TRIVIAL ZEROES). [+] Real (MAX = 90° ANGLES)

O = [0] TIME: MAY BE ABSORBED BY THE MATERIAL (THE ZERO POINT) (ZP). [=] MIRROR (MORPHIC FUNCTION)

P = [2] EVEN: MAY BE TRANSMITTED THROUGH THE SURFACE (THE TRIVIAL ZEROES). [-] //MAGE
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THE LAW OF **Reflection** ESTABLISHES THE **CRITICAL LINE** AT ½ **Re** (DUE NORTH) & **MIRROR IMAGE** (DUE SOUTH). **LIGHT** (OBEYING THE LAW OF **Reflection**) SHALL ALWAYS PLOT ON THE ½ (**CRITICAL LINE**).

QUANDRANT REFLECTION (MAXIMUM REFLECTION) = 90° ANGLES (IN-PHASE). Q IS REFLECTED ON RE PLANE AT 90° (IN-PHASE) & SYMMETRICALLY THROUGH THE MEROMORPHIC FUNCTION. LIGHT (Q) (THE PERFECT WAVE) SHALL ARC AT 90° ANGLES (IN-PHASE) ON PLOTTED ZETA FUNCTIONS [0]. LIGHT (Q) PRODUCES THE ROSE CARDIOD (LEMICON TRISECTRIX = A HYPERBOLA WITH ONE FOCAL POINT). THE RIEMANN ZETA FUNCTION = THE VISUAL GEOMETRIC PERFECT 90° BENDING OF LIGHT UPON 2-D EUCLIDIAN HYPERSPACE (AS ALL ZETA ANGLES = 90°).

Refraction: The **Refractive** index of a material is a dimensionless number that describes how fast **Light** travels through the material. It is defined as: $n = \frac{c}{v}$ where c is the speed of **Light** in a **Vacuum** & v is the phase velocity of **Light** in the medium. The law of **Refraction** (snell's law) is also satisfied in metamaterials, which allow **Light** to be bent "backward" at a **Negative** angle of **Refraction** with a **Negative Refractive** index (**Light** inversion).

DIFFRACTION: MAXIMUM **DIFFRACTION** = THE OPENING [0] IS SHORTER THAN THE WAVELENGTH [1].

TRI-POLARITY:

1D ELECTR-O-MAGNETIC TRI-POLARIZATION [- = +]:

- [+] POSITIVE (ELECTRICTY)
- [=] NEUTRAL (-O-)
- [-] NEGATIVE (MAGNETISM)

2D GEOMETRIC (LINEAR) TRI-POLARIZATION [- = +]:

- [+] 2D (EUCLIDIAN PLANE): LINEAR POLARIZATION = A VERTICAL DIPOLE ANTENNA WILL CREATE A VERTICAL, LINEARLY POLARIZED ELECTR-O-MAGNETIC WAVE.
- [=]: BOTH **0** (**Z**) (**UN**-POLARIZED) & **T** (**OMNI**-POLARIZED) (**SUNLIGHT** IS A HOMOGENEOUS MIXTURE OF **ELECTR-O-MAGNETIC WAVES** WITH ALL ORIENTATIONS (**OMNI**DIRECTIONAL).
- [-]: FARADAY EFFECT: LINEAR POLARIZED SIGNALS FROM SPACE ARE SUBJECT TO FARADAY ROTATION CAUSED BY THE EARTH'S MAGNETIC FIELD.

3D GEOMETRIC (CIRCULAR) TRI-POLARIZATION [- = +]:

[+]: RIGHT-HAND CIRCULAR POLARIZATION (RHCP) = CLOCKWISE ROTATION IN THE DIRECTION OF PROPAGATION.

[=]: **3**D (HYPERPLANE): **CIRCULAR** POLARIZATION (CORKSCREW) = THE <u>ELECTRIC FIELD ROTATES</u> AS IT TRAVELS ALONG. **CIRCULAR** POLARIZATION IS NOT AFFECTED BY FARADAY ROTATION.

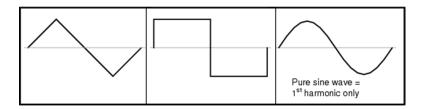
[-]: LEFT-HAND CIRCULAR POLARIZATION (LHCP) = COUNTER-CLOCKWISE ROTATION IN THE DIRECTION OF PROPAGATION.

THE FARADAY EFFECT IS CAUSED BY THE PROPERTY OF **CIRCULAR BIREFRINGENCE** (**LEFT & RIGHT CIRCULARLY** POLARIZED **WAVES** PROPAGATING AT SLIGHTLY DIFFERENT SPEEDS). SINCE A **LINEAR** POLARIZATION CAN BE DECOMPOSED INTO THE SUPERPOSITION OF **TWO EQUAL**-AMPLITUDE **CIRCULARLY** POLARIZED COMPONENTS OF OPPOSITE HANDEDNESS (**R**HCP [+] & **L**HCP [-]) & DIFFERENT PHASE, THE EFFECT OF A RELATIVE PHASE SHIFT, INDUCED BY THE FARADAY EFFECT, IS TO ROTATE THE ORIENTATION OF A WAVE'S **LINEAR** POLARIZATION. THE FAMILIAR ROTATION OF THE AXIS OF **LINEAR** POLARIZATION RELIES ON THE UNDERSTANDING THAT A **LINEARLY** POLARIZED **WAVE** CAN AS WELL BE DESCRIBED AS THE SUPERPOSITION (**ADDITION**) OF A **LEFT & RIGHT CIRCULARLY** POLARIZED **WAVE** IN **EQUAL** PROPORTION. THE PHASE DIFFERENCE BETWEEN THESE **TWO** WAVES IS DEPENDENT ON THE ORIENTATION OF THE **LINEAR** POLARIZATION: θ_0 & THEIR **ELECTRIC FIELDS** HAVE A RELATIVE PHASE DIFFERENCE OF $2\theta_0$ WHICH THEN **ADD** TO PRODUCE **LINEAR** POLARIZATION:

$$\mathbf{E}_{ heta_0} = rac{\sqrt{2}}{2}(e^{-i heta_0}\mathbf{E}_{RHC} + e^{i heta_0}\mathbf{E}_{LHC})$$

POLARIZATION AS A FUNCTION OF ROOT 2.

THE THREE PERFECT WAVEFORMS



WAVEFORMS SYMMETRIC ABOUT THEIR X-AXIS CENTER LINE CONTAIN ONLY ODD HARMONICS.

THE DISTINCTION BETWEEN A WAVEFORM HAVING EVEN HARMONICS VERSUS NO EVEN HARMONICS RESIDES IN THE DIFFERENCE BETWEEN A TRIANGLE WAVESHAPE & A SAWTOOTH WAVESHAPE. THAT DIFFERENCE IS SYMMETRY ABOVE & BELOW THE HORIZONTAL CENTERLINE OF THE WAVE. A WAVEFORM THAT IS SYMMETRICAL ABOVE & BELOW ITS CENTERLINE (THE SHAPE ON BOTH SIDES MIRROR EACH OTHER PRECISELY) & CONTAIN NO EVEN-NUMBERED HARMONICS. SQUARE WAVES, TRIANGLE WAVES, & PURE SINE WAVES ALL EXHIBIT THIS SYMMETRY, & ALL ARE DEVOID OF EVEN HARMONICS.

HARMONICS

FORMULA (FUNDAMENTAL FREQUENCY): $E = V_{max}(2\pi ft)$ THE HARMONIC VALUES WILL BE GIVEN AS:

SECOND HARMONIC: $\begin{aligned} &\textbf{E}_2 = \textbf{V}_{2\text{max}}(2*2\pi\text{ft}) = \textbf{V}_{2\text{max}}(4\pi\text{ft}), = \textbf{V}_{2\text{max}}(2\pi\text{t}) \\ &\textbf{THIRD HARMONIC:} &\textbf{E}_3 = \textbf{V}_{3\text{max}}(3*2\pi\text{ft}) = \textbf{V}_{3\text{max}}(6\pi\text{ft}), = \textbf{V}_{3\text{max}}(3\omega\pi\text{t}) \\ &\textbf{FOURTH HARMONIC:} &\textbf{E}_4 = \textbf{V}_{4\text{max}}(4*2\pi\text{ft}) = \textbf{V}_{4\text{max}}(8\pi\text{ft}), = \textbf{V}_{4\text{max}}(4\pi\text{t}) \end{aligned}$

...

THE HARMONICS OF MUSIC THEORY (THE SOUND OF THE HARMONY POINTS OF THE PERFECT SINEWAVE) ARE SYNCHRONOUS WITH THE HARMONIC SERIES EXIBITED BY THE RIEMANN ZETA FUNCTION. BOTH ARE SYNCHRONOUS WITH LIGHT (ELECTR-O-MAGNETIC CURRENT).

ZETA PLOTS	HARMONICS		FREQUENCIES	
(Z1 - Z34.1)	(12, 7, 5,)	$(432 \text{ Hz} = A_4)$	$(440 \text{ Hz} = A_4)$	$(444 \text{ Hz} = A_4) (37 \times 12)$
Z1: 14 .134725 14 2	7 x 2	,		. , , ,
Z2: 21.022039639	7 x 3	$21.43 = F_0$	$21.83 = F_0$	
Z3: 25.010857580	5 x 5	25.48 = G# ₀	25.96 = G# ₀	
Z4: 30.424876126	5 x 6	$30.31 = B_0$	$30.87 = B_0$	
Z5: 32.935061588	2 x 16	$32.11 = C_1$	$32.70 = C_1$	
Z6: 37.5 <mark>861</mark> 78159	37 x 3 = 111			
Z7: 40.918719012	5 x 8	$40.45 = E_1$		
Z8: 43 . 32 7073281	52 / 12 = 4.333			$43.65 = F_1$
Z9: 48.005150881	12 x 4	$48.11 = G_1$		
Z10: 49.773832478	7 x 7		$49.00 = G_1$	$49.44 = G_1$
Z11: 52.97032 14 78	4 x 13			52.39 = G# ₁
Z12: 56.446247697	7 x 8			
Z13: 59.3470 44 003	59 / 12 = 4.91666			
Z14: 60.831778525	12 x 5	$60.61 = B_1$		
Z15: 65.1125 44 048	5 x 13		$65.41 = C_2$	
Z16: 67.079810529	67 / 12 = 5.58333			
Z17: 69.546401711	3 x 23		$69.30 = C#_2$	$69.93 = C#_2$
Z18: 72.067157674	12 x 6	$72.08 = D_2$		
Z19: 75.704690699	5 x 15			
Z20: 77. 144 840069	7 x 11		$77.78 = D#_2$	
Z21: 79.337375020	79 / 12 = 6.58333			
Z22: 82.910380854	82 / 12 = 6.8333			$82.41 = E_2$
Z23: 84.735492981	12 x 7			
Z24: 87.425274613	3 x 29		$87.31 = F_2$	
Z25: 88.809 111 208	8 x 11			$88.10 = F_2$
Z26: 92.491899271	4 x 23		$92.50 = F#_2$	
Z27: 94.6513 44 04 1	94 / 12 = 7.8333 (SCH	IUMANN RESON	IANCE)	
Z28: 95.870634228	5 x 19			
Z29: 98.831194218	2 x 49 [7 x 7]		$98.00 = G_2$	$98.89 = G_2$
Z30: 101.317851006	101 / 3 = 33.666	$101.94 = G#_2$		
Z31: 103.725538040	103 / 3 = 34.333		$103.83 = G#_2$	
Z32: 105. 44 6623052				
Z33: 107. 1686111 84	•			
Z34: 111 .029535543				111 .00 = A_2
Z34.1: 111 .874659177.	3 x 37			111 .00 = A_2
111 = 37 x 03	444 = 37 x 12	777 = 37 x 21	1110 =	37 x 30
222 = 37 x 06	555 = 37 x 15	888 = 37 x 24	1221 =	37 x 33
333 = 37 x 09	666 = 37 x 18	999 = 37 x 27	1332 =	37 x 36 37 x 37 = 1369

SQUARE WAVE SIGNALS:

A SQUARE WAVE = A SINUSOIDAL (PERFECT CIRCLE) WAVE.

SQUARE WAVE = A CIRCULAR SINEWAVE AT THE SAME (FUNDAMENTAL) FREQUENCY ADDED TO AN INFINITE SERIES OF ODD-MULTIPLE SINEWAVE HARMONICS AT DECREASING AMPLITUDES.

ANY REPEATING, NON-SINUSOIDAL WAVEFORM (NO MATTER HOW STRANGE OR CONVOLUTED THE WAVEFORM IN QUESTION MAY BE) CAN BE EQUATED TO A COMBINATION OF DC VOLTAGE, SINEWAVES, &/OR COSINE WAVES (SINEWAVES WITH A 90-DEGREE PHASE SHIFT) AT VARIOUS AMPLITUDES & FREQUENCIES. ANY WAVE THAT REPEATS ITSELF REGULARLY OVER TIME IS REDUCIBLE TO THIS SERIES OF SINUSOIDAL WAVES. IN PARTICULAR, SQUARE WAVES ARE MATHEMATICALLY EQUIVALENT TO THE SUM OF A SINEWAVE AT THAT SAME FREQUENCY, PLUS AN INFINITE SERIES OF ODD-MULTIPLE FREQUENCY SINEWAVES AT DIMINISHING AMPLITUDE. THUS, A SQUARE WAVE IS AN INFINITE SERIES OF ODD-MULTIPLE FREQUENCY SINEWAVE SINEWAVE HARMONICS ADDED TOGETHER (LAW OF HARMONIC PROGRESSION):

1V PEAK REPEATING **SQUARE WAVE** AT **44** HZ IS **EQUIVALENT** TO:

MIRROR INVERSIONS OF THE LAW OF HARMONIC PROGRESSION:

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Leibniz Formula: \pi/4 = arctan (1) = 1 - 1^3/3 + 1^5/5 - 1^7/7 + 1^9/9 - ... (ad infinitum) ... Gregory's Formula: \pi/4 = 1 - 1/3 + 1/5 - 1/7 + 1/9 - ... (ad infinitum) ...
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CIRCULAR WAVE SIGNALS:

A CIRCULAR WAVE = A SINUSOIDAL (PERFECT CIRCLE) WAVE.

A WAVEFORM OF INFINITE DURATION (INFINITE NUMBER OF CYCLES) CAN BE ANALYZED WITH ABSOLUTE PRECISION, BUT THE LESS CYCLES AVAILABLE FOR ANALYSIS, THE LESS PRECISE THE ANALYSIS. IT IS ONLY WHEN WE HAVE AN EQUATION DESCRIBING A WAVEFORM IN ITS ENTIRETY THAT FOURIER ANALYSIS CAN REDUCE IT TO A DEFINITE SERIES OF SINUSOIDAL WAVEFORMS. THE FEWER TIMES THAT A WAVE CYCLES, THE LESS CERTAIN ITS FREQUENCY. TAKING THIS CONCEPT TO ITS LOGICAL EXTREME, A SHORT PULSE (A WAVEFORM THAT DOESN'T EVEN COMPLETE A CYCLE) ACTUALLY HAS NO FREQUENCY [0], BUT RATHER ACTS AS AN INFINITE RANGE OF FREQUENCIES [∞]. THIS PRINCIPLE APPLIES TO ALL WAVEBASED PHENOMENA (NOT JUST AC VOLTAGES & CURRENTS). SINCE DC DOES NOT ALTERNATE ITS "FREQUENCY" IS SAID TO BE ZERO [0].

FREQUENCY IS DEFINED AS A NUMBER OF CYCLES PER UNIT TIME (1 SECOND = 1 Hz). A FREQUENCY OF ½ Re PART IS PRODUCED (CYCLED) AT 2 SECONDS. INVERSELY PROPORTIONAL, A FREQUENCY OF 2 SHALL PRODUCE (CYCLE) A FREQUENCY OF ½ Re PART. ZETA PLOTS & HARMONIC FREQUENCY SYNCHRONIZE:

FREQUENCY	ZETA PLOTS
1 Hz = 1 s	1 = ∞
-1 Hz = -1 s	-1 = -1/12 (0.08333) (OR, -1 SCALED DOWN TO A 1:12 RATIO)
0 Hz = 0 s Π Hz = Π s	0 = PRIME ZERO (NO FREQUENCY =) (MÖBIUS INVERSION) $\mathbf{\Pi}$ = PRIME ZERO (ALL FREQUENCY)
2 Hz = ½ s ½ Hz = -2 s	2 = Π ² /6 (1.644934) (THE BASEL PROBLEM) ½ Re PART = ALL NON-TRIVIAL ZEROS, &; -2 (& ALL -EVEN INTEGERS) = ALL TRIVIAL ZEROS.

RIEMANN SHOWED THAT THE FUNCTION DEFINED BY THE DIRICHLET SERIES ON THE ½-PLANE OF CONVERGENCE CAN BE CONTINUED ANALYTICALLY TO ALL COMPLEX VALUES S \neq 1. FOR S = 1 THE SERIES IS THE HARMONIC SERIES WHICH DIVERGES TO $+ \infty$ (AFFINITY). THUS, THE RIEMANN **ZETA** FUNCTION IS A **MEROMORPHIC** FUNCTION ON THE WHOLE COMPLEX S-PLANE, WHICH IS HOLOMORPHIC EVERYWHERE EXCEPT FOR A SIMPLE POLE AT S = 1 (THE **FIRST** EMANATION OF Q (**LIGHT**)).

CONSTRUCTION OF THE 12-FOLD SINUSOIDAL WAVE:

THE INSTANTANEOUS VALUES OF A SINUSOIDAL WAVEFORM ARE GIVEN AS:

"The Instantaneous Value = Maximum Value x Sin θ "

Formula: $V_i = V_{max} x Sin \theta$ $V_{max} = \text{the Maximum Voltage Induced in the Coil.}$

 θ = ω t, = the **Ro**tational Angle of the Coil with **Re**spect to **Time**.

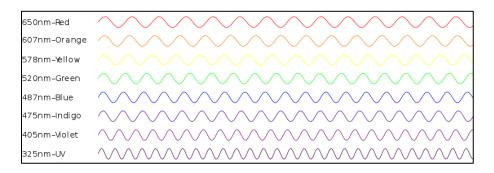
TO CONSTRUCT THE SINUSOIDAL WAVEFORM, CALCULATE THE INSTANTANEOUS VALUES AT VARIOUS POINTS ALONG THE WAVEFORM (USING THE MAXIMUM OR PEAK VALUE OF THE WAVEFORM). USE THE FORMULA ABOVE & PLOT THE INSTANTANEOUS VALUES EVERY 30° (12 POINTS) ON THE 2-D EUCLIDIAN PLANE (SQUARE GRAPH PAPER) OR 3-D FANO PLANE (TRIANGULAR & CUBIC GRAPH PAPER).

INTERFERENCE Formula: $\Delta \phi = 2\pi \Delta x/\lambda$

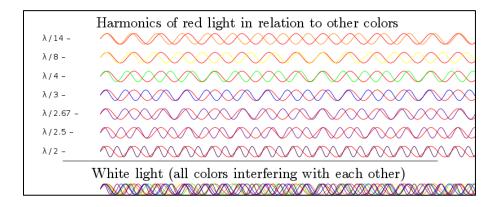
ALL ELECTR-O-MAGNETIC WAVES CAN BE SUPERIMPOSED UPON EACH OTHER WITHOUT LIMIT. THE ELECTRIC [+] & MAGNETIC [-] FIELDS [=] SIMPLY ADD AT EACH POINT.

[+] $\mathbf{0}^{0}$ - 120° = CONSTRUCTIVE INTERFERENCE (INCREASED STRENGTH) [=] 180° = TOTAL CONSTRUCTIVE INTERFERENCE (NO PHASE SHIFT = NO SIGNAL) = BOTH $\mathbf{0}$ (\mathbf{z}) & \mathbf{T} [-] 120° - 240° = DESTRUCTIVE INTERFERENCE (DECREASED STRENGTH)

LIGHT HARMONICS

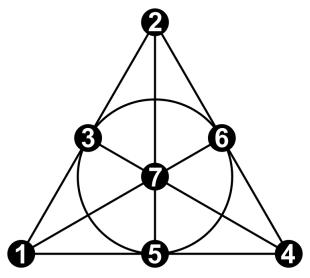


[½ Re PART & 1]: CHROMATICS (COLOR HARMONICS): THE HARMONIC-ANALYSIS CHART SHOWS HOW THE DIFFERENT WAVELENGTHS INTERACT WITH RED LIGHT. AT A DIFFERENCE OF $\lambda/2$ (½ Re PART WAVELENGTH), RED (POLE S 1) IS PERFECTLY IN SYNC WITH ITS SECOND HARMONIC IN THE ULTRAVIOLET (WHICH MEANS THE VISIBLE SPECTRUM (RAINBOW) IS CREATED BETWEEN ½ Re PART (THE CRITICAL LINE) TO POLE S 1).



[0 & 2]: OSCILLATIONS: ALL OTHER WAVELENGTHS IN THE VISUAL SPECTRUM HAVE LESS THAN A $\lambda/2$ DIFFERENCE BETWEEN THEM, FORMING HARMONIC OSCILLATIONS IN THE COMBINED WAVES. THE PICTURE SHOWS HOW THE $\lambda/4$ HARMONIC INTERACTS IN VISIBLE LIGHT (GREEN & RED), AS PHOTOGRAPHED IN AN OPTICAL FLAT. THE OSCILLATIONS ARE MOST RAPID AT $\lambda/4$ ($\lambda/2^2$), CYCLING EVERY 4TH WAVE, WHILE AT $\lambda/8$ ($\lambda/2^3$) THEY CYCLE EVERY 8TH WAVE.

[Π]: AT $\lambda/3$ THEY CYCLE EVERY 7TH WAVE, & AT $\lambda/2.5$ (2 & 5 ARE MIRROR INVERSIONS. REF: SCHUMANN RESONANCE) THEY CYCLE EVERY 13TH. AT $\lambda/14$, THEY CYCLE EVERY 14TH WAVE (x 6 = 84) (x 12 = 168).



A NIMBER NUMBERING OF THE FANO PLANE

THE 3D FANO PLANE CONFIGURATION IS SYNCHRONOUS WITH THE CHROMATIC OSCILLATION OF $\lambda/14$ (ORANGE), WHICH CYCLES EVERY 14TH WAVE (x 6 = 84) (x 12 = 168) WITH 7 = ½ Re PART.

CIRCULAR POLARIZED $\lambda/14$ (14TH WAVE) = 3 + 6 + 5 (7 DAYS)

LINEAR TRI-POLARIZED	MIRRORS
$\lambda/14$ (14TH WAVE) = 1 + 7 + 6	$\lambda/3$ (7TH WAVE) = 1 + 6 = 7
$\lambda/14$ (14TH WAVE) = 2 + 7 + 5	$\lambda/3$ (7TH WAVE) = 2 + 5 = 7
$\lambda/14$ (14TH WAVE) = 3 + 7 + 4	$\lambda/3$ (7TH WAVE) = 3 + 4 = 7

SQUARE POLARIZED $\lambda/14$ (14TH WAVE) = 1 + 2 + 4 + 7

A COLLINEATION, AUTOMORPHISM (SYMMETRY) OF THE FANO PLANE IS A PERMUTATION OF THE 7 POINTS THAT PRESERVES COLLINEARITY. THIS PROJECTIVE SPECIAL LINEAR GROUP PSL (2, 7) HAS 168 ELEMENTS & IS THE 2ND SMALLEST NON-ABELIAN GROUP. (168 / 7 = 28) & (1 + 2 + 3 + 4 + 5 + 6 + 7 = 28).